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Review Article

STUDY ON ABELIAN DOMINANCE OF INTERQUARK POTENTIAL IN SU (3) LATTICE QCD USING SIMULATION TECHNIQUES

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Abstract: We present study of the abelianization of SU(3) quenched lattice QCD in maximally abelian (MA) gauge in light of dual superconductor picture. After the MA gauge fixing, we studied the use of Cartan decomposition to split the gauge field in QCD into Abelian and off-diagonal parts. For the study of potentials related to these parts as V(r) for interquark potential, $V_{Abel}(r)$ for Abelian potential and $V_{off}(r)$ for off diagonal potential, we reviewed the lattice calculations of several size lattices by various authors at β=5.8-6.4 and observed relationship among these potentials which suggests a non trivial relation $V(r) = V_{Abel}(r) + V_{off}(r)$.

Index Terms – Quark confinement, Dual superconductor picture, Abelian projection, MA gauge.

Introduction: The simulation calculations are made at a large distance scale to understand the phenomenon of quark confinement. Monte Carlo simulation on QCD lattice show that quark potential increases linearly at large distances (beyond the distance of few fermies) and the strength of potential is controlled by the string tension σ. However for small distances the quark-antiquark $Q\bar{Q}$ potential V(r) behaves as coulomb potential.
Mathematically interquark potential V(r) is given as [1-3]

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$$V(r) = -\frac{A}{r} + \sigma r + C \qquad (1)$$

where, r is the interquark distance, and σ is the string tension, A is color-Coulomb coefficient and C is an irrelevant constant.
Lattice QCD study gives that the linear confinement term arises due to the one dimensional color flux squeezing of the quarks [1]. In 1970's Nambu, t'Hooft and Mandelstam [4] presented the concept of dual superconducting picture. From the viewpoint of the dual superconducting picture Ginzburg Landu theory [5] was formulated, and described the flux tube structure of hadrons.
Dual Superconductor picture: The dual superconductivity is the promising scenario for quark confinement. Dual superconductivity is the electromagnetic duality of the ordinary superconductivity. As superconductivity is

caused by the condensation of cooper pair (electrically charged particles) similarly the dual superconductivity is assumed to be caused by the condensation of magnetic monopoles (magnetically charged particles). In dual superconductors the dual Meissner effect squeezes the color electric flux between the quarks in the form of tube like region as shown in the Figure 1.

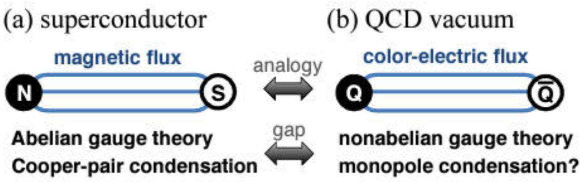

Fig. 1 Dual Superconducting picture

The dual superconducting picture provides a framework for the study of quark confinement however there are two major drawbacks which make it difficult to define QCD vacuum from this picture. These drawbacks are as given below:

Drawbacks of Dual Superconducting picture: (1).The dual superconducting picture supports to the Abelian dominance i.e. dual superconductivity is assumed as the Abelian U(1) gauge theory like QED, while QCD is a non abelian SU(3) gauge theory.

(2).The dual superconductivity considers the existence of the electromagnetic duality of cooper pairs, but there is no resemblance of monopole condensation with QED.

To establish the dual superconducting picture it is essential to overcome these drawbacks. To resolve these gaps currently there are at least two methods available.

Abelian Projection: Abelian Projection is a mathematical procedure has been proposed by t'Hooft [6] through which the confining physics of an SU (N) gauge theory can be explained by the $U(1)^{N-1}$ degrees of freedom. In particular of SU (3) QCD, it is possible to reduce QCD to an Abelian U (1) X U (1) gauge theory including monopole degrees of freedom.

Field decomposition and change of variable: This method has been proposed by Cho [7] and Duan and Ge [8] and latest by Faddeev and Niemi [9] and Shabanov [10].So it is called CDGFNS decomposition. In this method the monopole condensation is defined by gauge independent way. Abelian projection can be regarded as gauge fixed version of field decomposition.

In our review study we are focused on Abelian projection which is an extensively used method to extract magnetic monopoles from Yang Mills field, refers that only diagonal gluon components play the dominant role for the non perturbative QCD phenomenon like confinement.

Maximally Abelian Gauge (MAG) and Cartan Decomposition: The maximally abelian (MA) gauge has been successfully applied for the analysis of gluon propagator [11],to explain large effective mass generation of off diagonal gluons and for the study of infrared abelian dominance[12].In continuum Euclidean QCD the MA gauge is maximized by minimizing the off diagonal gluon field under the gauge transformation [11-13].

The gluon field A_μ in QCD has both diagonal and off diagonal parts. In continuum, Cartan decomposition is given by –

$$A_\mu = \vec{A}_\mu.\vec{H} + \sum_\alpha A_\mu^\alpha T_\alpha \qquad (2)$$

where $\vec{H}= T_3, T_8$ are the diagonal generators, and $T_{\alpha=1,2,4,5,6,7}$ are the off diagonal generators of SU(3).The off diagonal part $\sum_\alpha A_\mu^\alpha T_\alpha$ represent the non abelian nature and $A_\mu \rightarrow \vec{A}_\mu.\vec{H}$ induces the Abelian Projection. For MA projection the minimization of off diagonal part is given as-

$$R_{off}[A_\mu(x)] \equiv \int d^4x \sum_{\mu,\alpha} \left|A_\mu^\alpha(x)\right|^2 \qquad (3)$$

The SU(3) QCD link variables $U_\mu(s)$ is given as-

$$U_\mu(s) = e^{iagA_\mu(s)} \in SU(3) \quad (4)$$

where 'a' is the lattice spacing and 'g' is gauge coupling. In lattice formalism MA gauge is defined by maximizing Abelian part of link variables under the SU(3) gauge transformation.

$$R_{MA}[U_\mu(s)] \equiv \sum_s \sum_{\mu=1}^{4} Tr(U_\mu^\dagger(s)\vec{H}U_\mu(s)\vec{H}] \quad (5)$$

After the MA gauge fixing ,the SU(3) link variables are factorized with respect to the Cartan decomposition of SU(3) into $U(1)^2$.The SU(3) gauge transformation of link variable is defined by-

$$U_\mu(s) \to U_\mu^w(s) \equiv w(s)U_\mu(s)w^\dagger(s+\hat{\mu}) \text{ with } w(s) \in SU(3) \quad (6)$$

$U_\mu^{MA}(s) \in SU(3)$ be the link variables in MA gauge .The abelian part of the link variable is extracted as-

$$u_\mu(s) = \exp\,(i\theta_\mu(s).H) \in U(1)^2 \quad (7)$$

also may be written as,

$$u_\mu(s) = \exp\,(i\theta_\mu^3(s)T_3 + i\theta_\mu^8(s)T_8) \in U(1)_3 \times U(1)_8 \quad (8)$$

and the off diagonal link variable is defined as-

$$M_\mu(s) = U_\mu^{MA}(s)u_\mu^\dagger(s) \in SU(3)/U(1)_3 \times U(1)_8 \quad (9)$$

which leads to the Cartan decomposition of the gauge group

$$U_\mu^{MA}(s) = M_\mu(s)u_\mu(s)$$

$$= e^{i\sum\theta_\mu^\alpha T_\alpha(s)} e^{i\theta_\mu(s).H} \in SU(3) \quad (10)$$

In the MA gauge, there remains residual $U(1)^2$ Abelian gauge symmetry, which does not change the MA gauge condition. Using the Abelian gauge function $\upsilon(s) \in U(1)^2$

$$U_\mu(s) \to U_\mu^\upsilon(s) \equiv \upsilon(s)U_\mu(s)\upsilon^\dagger(s+\hat{\mu}) \quad (11)$$

The off diagonal link variable behave as charged matter fields to keep the form of $SU(3)/U(1)^2$.

$$M_\mu(s) \to M_\mu^\upsilon(s) \equiv \upsilon(s)M_\mu(s)\upsilon^\dagger(s) \in SU(3)/U(1)^2 \quad (12)$$

Abelian link variables behave as abelian gauge field.

$$u_\mu(s) \to u_\mu^\upsilon(s) \equiv \upsilon(s)\,u_\mu(s)\upsilon^\dagger(s+\hat{\mu}) \quad (13)$$

Numerical conditions and Formalism for $Q\bar{Q}$ Potential Calculations: We review the numerical analysis performed by N. Sakumichi et.al [14] for $Q\bar{Q}$ potential calculations. The Monte Carlo simulation was performed using SU(3) standard plaquette action with lattice parameter $\beta = 5.8 - 6.4$,and various lattices of spatial size La$\sim$ $2-3fm$ corresponding to the lattice spacing a=0.058-0.148 fm. Large no. of gauge configurations (200-600) were sampled. For MA gauge fixing overrelaxation method was used. Also smearing method was used for accurate measurement.

For a closed R×T rectangle trajectory C, the Wilson loop $W_C[U_\mu(s)] \equiv Tr[\prod_C U_\mu(s)]$.The quark-antiquark potential was calculated as-

$$V(r) = -\lim_{T\to\infty} \frac{1}{T} ln\langle W_C[U_\mu(s)]\rangle \quad (14)$$

where $\langle ... \rangle$ is the statistical average over the gauge configuration .The MA projection of the quark-antiquark potential was calculated as-

$$V_{Abel}(r) = -\lim_{T\to\infty} \frac{1}{T} ln\langle W_C[u_\mu(s)]\rangle \quad (15)$$

The off diagonal part of the quark-antiquark potential was calculated as-

$$V_{off}(r) = -\lim_{T\to\infty} \frac{1}{T} ln\langle W_C[M_\mu(s)]\rangle \quad (16)$$

Quantitative Analysis on Abelian Dominance for confinement: Sakumichi *et al.* [14] calculated the SU(3) potential V(r) and Abelian projected potential $V_{Abel}(r)$ in MA gauge at $\beta = 6.0 - 6.4$ on 32^4 lattice and $\beta = 5.8$ on $16^3 32$ lattice. Fit analysis taken from Table I with Coulomb plus linear ansatz of Eq. 1.

	$32^4, \beta = 6.4$			$32^4, \beta = 6.0$			$16^3 32, \beta = 5.8$		
	σ	A	C	σ	A	C	σ	A	C
V	0.01528(12)	0.265(3)	0.598(1)	0.0471(4)	0.290(7)	0.659(4)	0.0988(19)	0.315(25)	0.679(15)
$V_{\rm Abel}$	0.01550(06)	0.056(1)	0.167(1)	0.0475(2)	0.044(3)	0.178(2)	0.0988(08)	0.039(10)	0.183(06)
$V_{\rm off}$	−0.00037(03)	0.175(1)	0.391(1)	−0.0009(1)	0.139(3)	0.415(1)			
$V - V_{\rm Abel}$	−0.00024(11)	0.209(3)	0.432(1)	−0.0005(3)	0.247(6)	0.481(3)	−0.0010(17)	0.285(21)	0.502(12)
$V - V_{\rm Abel}$	0	0.205(1)	0.429(1)	0	0.240(3)	0.476(1)	0	0.273(09)	0.494(03)
$V_{\rm Abel} + V_{\rm off}$	0.01528(07)	0.227(2)	0.556(1)	0.0459(2)	0.201(4)	0.602(2)			

Table I *Phys.Rev.D90, 11501(R) (2014)* represents Fit analysis with the Coulomb plus linear ansatz for QQ potentials.For each potential value of lattice parameters σ,A,C is listed in the functional form of eq.1

Fig. 2(a) reflects that at large distances there is resemblance in the behaviour of V(r) and $V_{Abel}(r)$. The same slope of V(r) and $V_{Abel}(r)$ suggests the Abelian dominance for confinement. To demonstrate the perfect abelian dominance N. Sakumichi *et al.* observed the behaviour of V(r) $-V_{Abel}(r)$ with interquark distance. In Fig.2 (b) the fit (1) is Coulomb plus linear ansatz and fit 2 is the pure Coulomb ansatz of Eq.1. From Fig.2 (b) and Table I , it appears that V(r) $-V_{Abel}(r)$ is well described by pure Coulomb ansatz and have almost zero string tension $\sigma \sim 0$.It may be understand as $\sigma_{su(3)} - \sigma_{abel} \sim 0$.It implies that $\sigma_{su(3)} \sim \sigma_{abel}$.This also suggests perfect abelian dominance for confinement.

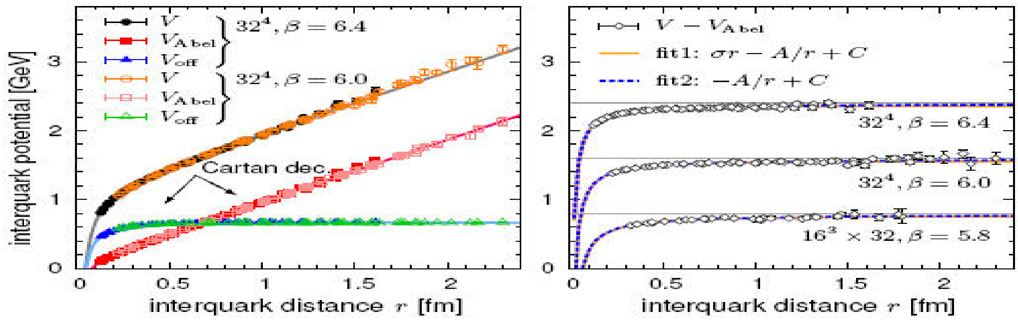

FIG. 2: (a) Cartan decomposition of the Q¯Q potential (b) Fit analysis of V (r) − V_{Abel} (r) to illustrate the perfect Abelian dominance of quark confinement. *Phys.Rev.D90, 11501(R) (2014)*

For the smaller lattices only approximate Abelian dominance was observed. Stack el.al [13] reported $\sigma_{abel}/\sigma \sim 0.90$ for 16^4 lattice at $\beta = 6.0$ and $\sigma_{abel}/\sigma \sim 0.94 - 0.95$ for $16^3 32$ lattice at $\beta = 6.2$. Fig. 3(a) shows that Perfect Abelian dominance seems to be realized when spatial size La is sufficiently large [14].

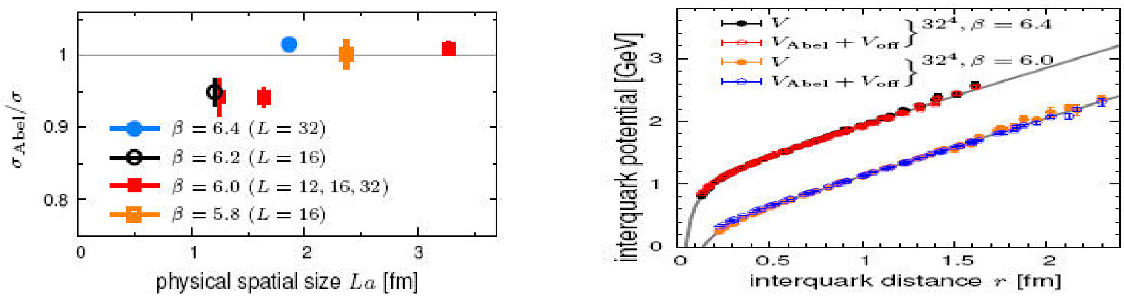

FIG.3: (a) Represent Physical spatial-size dependence of σ_{abel}/σ (b) Comparison between V(r) and $V_{Abel}(r) + V_{off}(r)$.*Phys.Rev.D90, 11501(R) (2014)*

From the analysis of V(r), $V_{Abel}(r)$ and $V_{off}(r)$ a summation relation $V(r) = V_{Abel}(r) + V_{off}(r)$ has been observed[14]. Fig.3(b) shows the comparison between $V(r)$ and $V_{Abel}(r) + V_{off}(r)$ for 32^4 lattice.The summation relation looks fair for large distances as well as small distances because the difference between V(r) and $V_{Abel}(r)$ is complemented by $V_{off}(r)$.But the summation formula is non trivial because of non abelian nature of gauge fields.

Consider $W_C[U_\mu(s)]$ is SU(3) Wilson loop as discussed in Eq. 14

$$W_C[U_\mu(s)] = \mathrm{Tr}[\textstyle\prod_C[U_\mu(s)]]$$
$$= \mathrm{Tr}[\textstyle\prod_C[M_\mu(s)u_\mu(s)]]$$

because the Abelian link variables $u_\mu(s)$ and off-diagonal link variables $M_\mu(s)$ are not commutable link variables, so we may write,

$$\mathrm{Tr}[\textstyle\prod_C[M_\mu(s)u_\mu(s)]] \neq \mathrm{Tr}[\prod_C[M_\mu(s)]].\ \mathrm{Tr}[\prod_C[u_\mu(s)]]$$

Then a simple factorization of Wilson loop and a simple summation on the potential cannot be expected.

In general, $W_C[U_\mu(s)] \neq W_C[M_\mu(s)]W_C[u_\mu(s)]$ and $V(r) \neq V_{Abel}(r) + V_{off}(r)$ (17)

This represents the non triviality of summation formula.

Conclusion: SU(3) potential and its abelian part have almost same slope at large distances. It refers the Abelian dominance of confinement. Perfect Abelian Dominance of string tension is analysed mathematically as $\sigma_{abel}/\sigma = 1$, seems to be realize when the spatial volume of the lattice is large (approximately greater than $(2\ fm)^3$).From the simulation of potentials V(r), $V_{Abel}(r)$ and $V_{off}(r)$ a simple, non trivial relation $V(r) = V_{Abel}(r) + V_{off}(r)$ is observed.

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References

[1] H. J. Rothe, Lattice Gauge Theories, 4th ed. (World Scientific, Singapore, 2012)
[2] G. S. Bali and K. Schilling, Phys. Rev. D 47, 661 (1993)
[3] T. T. Takahashi, H. Matsufuru, Y. Nemoto, and H. Suganuma, Phys. Rev. Lett. 86, 18 (2001); T. T. Takahashi,H. Suganuma, Y. Nemoto, and H. Matsufuru, Phys. Rev.D 65, 114509 (2002).
[4] Y. Nambu, Phys. Rev. D 10, 4262 (1974); G.'t Hooft, in High Energy Physics; S. Mandelstam, Phys. Rep. 23, 245 (1976)
[5] S. Maedan and T. Suzuki, Prog. Theor. Phys. 81, 229(1989); H. Suganuma,S. Sasaki, and H. Toki, Nucl. Phys.B435, 207 (1995)
[6] G. 't Hooft, Nucl. Phys. B190, 455 (1981)
[7] Y.M. Cho, Restricted gauge theory, Phys. Rev. D**21**, 1080–1088 (1980).
[8] Y.S. Duan and M.L. Ge, Sinica Sci., **11**, 1072–1081 (1979)
[9] L. Faddeev and A.J. Niemi, Partially dual variables in SU(2) Yang-Mills theory, Phys. Rev. Lett. **82**, 1624–1627 (1999)
[10] S.V. Shabanov, An effective action for monopoles and knot solitons in Yang-Mills theory, Phys. Lett. B**458**, 322–330(1999)
[11] K. Amemiya and H. Suganuma, Phys. Rev. D 60, 114509(1999)
[12] Shinya Gongyo, and Hideo Suganuma, Takumi Iritani, Phys. Rev.D 86(2012)094018, Phys. Rev.D 87(2013)074506
[13] J. D. Stack, S. D. Neiman, and R. J. Wensley, Phys. Rev.D 50, 3399 (1994).
[14] Naoyuki Sakumichi and Hideo Suganuma, Phys.Rev.D90, 11501(R) (2014)