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Original Research Article

USING THE PERTURBATION DUE TO OBLATENESS OF THE THIRD ORDER TO DECREASE THE EFFECT OF THE PERTURBATION DUE TO AIR DRAG ON LOW EARTH ORBIT SATELLITES

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Abstract: In this paper the combined effects of the perturbation due to secular oblateness up to third order and drag force on the semi-major axis and the eccentricity of LEO – satellites are considered. The purpose of the work is to decrease the resultant perturbation on these parameters. Numerical results are obtained for some critical orbits where these perturbations work against each other.

Key words: Oblateness, Air drag, Low Earth orbit satellites.

Introduction: The equation of motion for artificial satellites has the attraction of the Earth as the main term; the main perturbation on the satellites is the effect of the Earth's oblateness. Thus we need to discuss this oblateness and how to deal with it.

Many analytical treatments deal with that problem of a satellite motion under the oblateness effect ; Brouwer[1](1959), Kozai[2](1962), Kaula[3] (1966), Deprit and Rom[4] (1970), Berger[5] (1975), Kinoshita[6] (1977), Chazov[7](1988), Emelyanove[8] (1991) studied this problem of different kind of approximation but up to the second order; where there is no perturbation in the shape of the orbit i.e. there is no secular perturbations in the semi major axis and on the eccentricity. Nasonova[9] (1971), Tamarove[10] (1985), M.K.Ammar et.al.[11](2012) deal with the perturbation due to Earth's oblateness up to the third order which gives the secular perturbation due to oblateness on both the semi major axis and the eccentricity.

Also satellites are subjected to disturbing forces which are known as non-gravitational forces. These non-gravitational forces are, for example, atmospheric drag, solar radiation pressure,

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electromagnetic radiations, and so on. Aside from the effects of the Earth's oblateness, the largest perturbative force on law earth orbit (LEO) satellites is caused by the atmospheric drag. Any satellite passes within an altitude of about 800 Km is subjected to a perturbative force caused by the motion of the satellite through the Earth's atmosphere.

The acceleration due to air drag causes undesirable changes in the shape of the orbit and a continuous loss of the kinetic energy of the satellite. Thus an elliptical orbit will change to become circular, while circular orbit will change to become more nearly circular and decay.

The objective of this paper is not only to determine the perturbations due to oblateness and air drag in order to take them into account in the satellite maintenance operations, but also to use them versus each other to decrease the resultant perturbation on the satellite in some specific orbits during a specific time.

Materials and Methods

The effect of the perturbation due to air drag on the semi-major axis and the eccentricity of the satellite can be written as [12]

$$\left(\frac{da}{dt}\right)_D = -n a^2 \frac{A}{m} \rho C_D \frac{\left(1+2e\cos f + e^2\right)^{\frac{3}{2}}}{\left(1-e^2\right)^{\frac{3}{2}}} \qquad \ldots\ldots(1)$$

$$\left(\frac{de}{dt}\right)_D = -n a \frac{A}{m} \rho C_D \left(\frac{e+\cos f}{\sqrt{1-e^2}}\right) \sqrt{1+e^2+2e\cos f} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

And

$$\rho = \rho_0 e^{[-(\eta-\eta_0)/H]} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Where

- a The satellite semi-major axis,
- e The satellite eccentricity,
- A The satellite's average cross sectional area,
- m The satellite mass,
- n The mean motion,
- f The true anomaly,
- C_D The aerodynamic drag coefficient ,
- η The satellite altitude at any time,
- η_0 The satellite altitude at the perigee,
- ρ The density of the air at altitude η,
- ρ_0 The density of the air at the standard altitude,
- H The scale height,

Elimination of the short period terms from equations (1), (2) as follow

$$\left(\frac{da}{dt}\right)_D = -\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{\mu B}{a\,n}\right)\left(\frac{\left(1+2e\cos f+e^2\right)^{\frac{3}{2}}}{\left(1+e\cos f\right)^2}\right)df$$

$$\left(\frac{de}{dt}\right)_D = -\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{\mu B}{na^2}\right)\left(e+\cos f\right)\sqrt{1+e^2+2e\cos f}\left(\frac{\left(1-e^2\right)}{\left(1+e\cos f\right)^2}\right)df$$

Finally the secular perturbation on the semi-major axis and the eccentricity due to air drag up to the 6th order in eccentricity can be formulated in the form

$$\Delta_D a = -na^2\frac{A}{m}\rho C_D\left(1+\frac{3}{4}e^2+\frac{21}{64}e^4+\frac{55}{256}e^6\right)t \quad \ldots.(4)$$

$$\Delta_D e = -n a \frac{A}{m}\rho C_D\left(\frac{1}{2}e-\frac{5}{16}e^3-\frac{9}{128}e^5\right)t \quad \ldots\ldots(5)$$

Where $\Delta_D a$ and $\Delta_D e$ are the secular perturbation in semi major axis and eccentricity due to air drag respectively. The perturbations on the semi major axis and the eccentricity due to oblateness is of the form [12]

$$\begin{aligned}\Delta_3 a = & \frac{J_2 D_4\ R^6 n\ t}{a^5\left(1-e^2\right)^{21/2}}\left(\alpha_1+\alpha_2\cos(2i)+\alpha_3\cos(4i)+\alpha_4\cos(6i)\right) \\ & +\frac{J_2^3\ R^6 n\ t}{a^5\left(1-e^2\right)^{21/2}}\left(\alpha_5+\alpha_6\cos(2i)+\alpha_7\cos(4i)+\alpha_8\cos(6i)\right) \\ & +\frac{J_2^3 R^6 n^2 t^2}{a^5\left(1-e^2\right)^{21/2}}\Big[\left(\alpha_9+\alpha_{10}\cos(2i)+\alpha_{11}\cos(4i)+\alpha_{12}\cos(6i)\right) \\ & \qquad +\left(\alpha_{13}+\alpha_{14}\cos(2i)+\alpha_{15}\cos(4i)+\alpha_{16}\cos(6i)\right)\cos 2\omega \\ & \qquad +\left(\alpha_{17}+\alpha_{18}\cos(2i)+\alpha_{19}\cos(4i)+\alpha_{20}\cos(6i)\right)\cos 4\omega\Big]\end{aligned} \tag{6}$$

$$\begin{aligned}\Delta_3 e = & \frac{J_2 D_4\ R^6 n t}{a^6\left(1-e^2\right)^{19/2}}\left(\beta_1+\beta_2\cos(2i)+\beta_3\cos(4i)+\beta_4\cos(6i)\right) \\ & +\frac{J_2^3\ R^6 n t}{a^6\left(1-e^2\right)^{19/2}}\left(\beta_5+\beta_6\cos(2i)+\beta_7\cos(4i)+\beta_8\cos(6i)\right) \\ & -\frac{J_2 J_3\ R^5 n^2 t^2}{a^5\left(1-e^2\right)^{19/2}}\left(\beta_9\sin(i)+\beta_{10}\sin(3i)+\beta_{11}\sin(5i)\right)\sin(\omega) \\ & +\frac{J_2^3 R^6 n^2 t^2}{a^6\left(1-e^2\right)^{19/2}}\Big[\beta_{16}+\beta_{17}\cos(2i)+\beta_{18}\cos(4i)+\beta_{19}\cos(6i) \\ & \qquad +\left(\beta_{20}+\beta_{21}\cos(2i)+\beta_{22}\cos(4i)+\beta_{23}\cos(6i)\right)\cos(2\omega) \\ & \qquad +\left(\beta_{24}+\beta_{25}\cos(2i)+\beta_{26}\cos(4i)+\beta_{27}\cos(6i)\right)\cos(4\omega)\Big] \\ & +\frac{J_2 D_4\ R^6 n^2 t^2}{a^6\left(1-e^2\right)^{19/2}}\left(\beta_{12}+\beta_{13}\cos(2i)+\beta_{14}\cos(4i)+\beta_{15}\cos(6i)\right)\cos(2\omega)\end{aligned} \tag{7}$$

Where

μ The Newtonian constant times mass of Earth ,

R The mean equatorial radius of the Earth

ω The argument of the perigee,

J_k The zonal harmonics, given by

$$J_2 = 1.08263\times10^{-3},\ J_3 = -2.5322\times10^{-6},\ J_4 = -1.6110\times10^{-6},\ D_4 = J_2^2 + J_4 = -0.4389122\times10^{-6}$$

α_i *and* β_j , $i = 1,\ldots 20$; $j = 1,\ldots 27$; are functions of the eccentricity e and are given in Appendix A.

Therefore the resultant disturbing force due to air drag and due to oblateness on both the semi major axis and eccentricity are of the form

$$\Delta a = \Delta_D a + \Delta_3 a$$
$$\Delta e = \Delta_D e + \Delta_3 e \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

Where $\Delta\, a$ and $\Delta\, e$ are the resultant disturbing force due to air drag and due to oblateness on both the semi major axis and eccentricity respectively.

Results : Equations (8) clearly represent the resultant disturbing force due to air drag and due to oblateness on both the semi major axis and eccentricity. It can be used in satellites maintenance operations, but it can be used to decrease these operations too. Decreasing the maintenance operations here mean that decreasing the resultant disturbing forces acting on the satellite.

In order to do this we put equations (8) equals zero and solve them simultaneously in semi major axis and eccentricity for any particular satellite in a limited time so that we can get an orbit for this satellite where the resultant disturbing forces acting on it is zero i.e. solve the system using a satellite common physical date

$$\Delta a = 0$$
$$\Delta e = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

Table 1. Satellite physical data

Parameter	Value	Units
mass	175	[kg]
Area	2.22	[m^2]
C_d	2.3	[-]

Putting the argument of perigee $\omega = 0^0$ and calculating the orbital parameters of the desired orbit for 60 days we get

Table 2. The orbital parameters where the perturbation due to air drag and that due to oblateness work against each other.

Time (days)	Inclination (Degree)	Semi major axis (km)	Perigee (Km)	Eccentricity
60	24	6992.689006	599.992562	0.002078

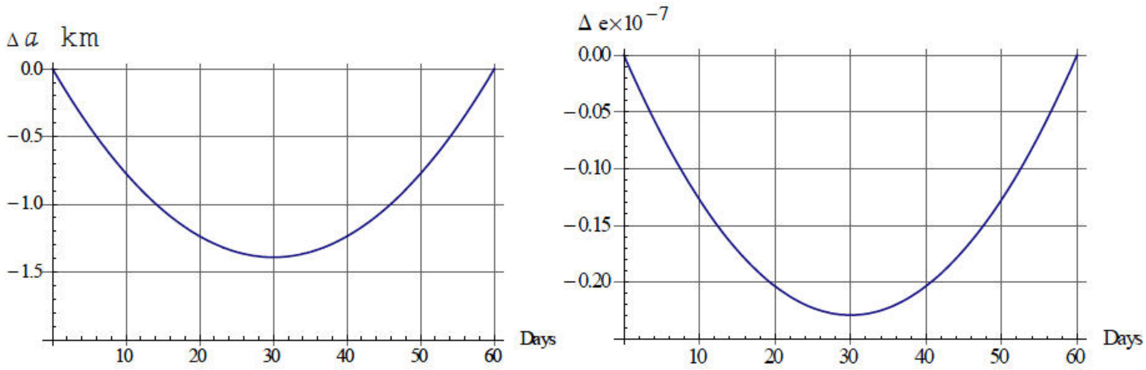

Figure 1 The resultant perturbation in the semi major axis and eccentricity during 60 days.

Conclusion

In this paper, we studied the resultant disturbing force acting on low earth orbit satellites due to air drag and oblateness on both the semi major axis and eccentricity; we put a simple formula to determine them for any low earth orbit satellite, so that the maintenance operation for this very important type of satellite can be determined in a very easy and accurate way.

Also, we put a method to cancel the resultant perturbations due to air drag and oblateness on both the semi major axis and eccentricity for a valuable duration of time. This method is very important for two reasons

1- This type of satellites has a very short life time, so any additional time to its life time will be great.

2- The second reason is the additional lifetime is free of cost since it comes from using natural forces against each other.

We apply our results in some particular satellite and we find the desired orbit for 60 days. i.e. we add 60 days to this satellite lifetime with no cost.

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Appendix A

$$\alpha_1=\frac{1755}{16384}e^2(8+46e^2+11e^4),\qquad \alpha_2=\frac{8955e^2}{32768}(8+46e^2+11e^4),$$

$$\alpha_3=\frac{2925e^2}{16384}(8+46e^2+11e^4),\qquad \alpha_4=\frac{4725e^2}{32768}(8+46e^2+11e^4),$$

$$\alpha_5=\frac{9}{262144}(-512+512e-525808e^2+450416e^3+695544e^4+1325848e^5+3541729e^6),$$

$$\alpha_6=-\frac{9}{524288}(-15872+15872e-54544e^2+3798288e^3+19551624e^4+12273896e^5+23257663e^6),$$

$$\alpha_7=\frac{9}{262144}(-11776+11776e-989328e^2+1282320e^3+3807816e^4+2972968e^5+6674807e^6),$$

$$\alpha_8=-\frac{9}{524288}(-8704+8704e-837616e^2+1068016e^3+4417784e^4+2930584e^5+6832241e^6),$$

$$\alpha_9=-\frac{9}{524288}(90944-95424e-283632e^2-283296e^3-697384e^4+300024e^5+1051709e^6),$$

$$\alpha_{10}=\frac{9}{1048576}(338112-614208e-3533584e^2-2112864e^3-1476568e^4+1808136e^5+6758243e^6),$$

$$\alpha_{11}=-\frac{9}{524288}(232128-253248e-1026704e^2-549984e^3-834968e^4+831816e^5+2419387e^6),$$

$$\alpha_{12}=\frac{9}{1048576}(209728-181440e-881136e^2-487584e^3-904104e^4+566136e^5+2365581e^6),$$

$$\alpha_{13}=\frac{9}{131072}(6144-20736e-57920e^2-44160e^3-46768e^4+66000e^5+119985e^6),$$

$$\alpha_{14}=-\frac{9}{262144}(2688-9600e-90880e^2-61824e^3-21976e^4+23496e^5+143303e^6),$$

$$\alpha_{15}=\frac{9}{131072}(1536-5376e-41664e^2-26496e^3-1072e^4+14544e^5+50311e^6),$$

$$\alpha_{16}=-\frac{27}{262144}(4224-14208e-36096e^2-26496e^3-24568e^4+45864e^5+65763e^6),$$

$$\alpha_{17}=-\frac{81}{524288}e^2(-152-160e-360e^2-272e^3+285e^4);$$

$$\alpha_{18}=\frac{27}{1048576}e^2(-3256-3680e-3080e^2-1456e^3+8185e^4),$$

$$\alpha_{19}=-\frac{27}{524288}e^2(-1784-2080e-520e^2+304e^3+5009e^4),$$

$$\alpha_{20}=\frac{81}{1048576}e^2(-408-480e-40e^2+144e^3+1181e^4),$$

$\beta_1 = \frac{1755}{32768} e(8 + 46e^2 + 11e^4)$, $\beta_2 = \frac{8955}{65536} e(8 + 46e^2 + 11e^4)$,

$\beta_3 = \frac{2925}{32768} e(8 + 46e^2 + 11e^4)$, $\beta_4 = \frac{4725}{65536} e(8 + 46e^2 + 11e^4)$,

$$\beta_5 = \frac{9}{524288} e(-309296 + 226352e - 269192e^2 + 1259352e^3 + 3797113e^4 + 829302e^5),$$

$$\beta_6 = -\frac{9}{1048576} e(789296 + 2682704e + 14989192e^2 + 12290600e^3 + 25356103e^4 + 7530674e^5),$$

$$\beta_7 = \frac{9}{524288} e(-402512 + 793168e + 1369288e^2 + 3106024e^3 + 7978655e^4 + 1108282e^5),$$

$$\beta_8 = -\frac{9}{1048576} e(-74800 + 757168e + 2173688e^2 + 3047000e^3 + 7852265e^4 + 1416366e^5),$$

$\beta_9 = \frac{117}{8192}(-16 + 88e^2 - 198e^4 + 231e^6)$, $\beta_{10} = \frac{315}{16384}(-16 + 88e^2 - 198e^4 + 231e^6)$,

$\beta_{11} = \frac{225}{16384}(-16 + 88e^2 - 198e^4 + 231e^6)$, $\beta_{12} = -\frac{855}{16384} e(8 - 36e^2 + 63e^4)$,

$\beta_{13} = -\frac{855}{32768} e(8 - 36e^2 + 63e^4)$, $\beta_{14} = \frac{495}{16384} e(8 - 36e^2 + 63e^4)$,

$\beta_{15} = \frac{1575}{32768} e(8 - 36e^2 + 63e^4)$,

$$\beta_{16} = -\frac{9}{1048576e}(26944 - 40128e + 22288e^2 - 304032e^3 - 1001128e^4 + 136440e^5 + 895805e^6 + 390132e^7),$$

$$\beta_{17} = \frac{9}{2097152e}(40640 - 328512e - 1988368e^2 - 2340960e^3 - 3379288e^4 + 1218312e^5 + 6697091e^6 + 2810124e^7),$$

$$\beta_{18} = -\frac{9}{1048576e}(50368 - 124224e - 180624e^2 - 695136e^3 - 1889048e^4 + 654408e^5 + 2513467e^6 + 538092e^7),$$

$$\beta_{19} = \frac{9}{2097152e}(15680 - 98496e - 122352e^2 - 591264e^3 - 1717032e^4 + 473976e^5 + 2303085e^6 + 539316e^7),$$

$$\beta_{20} = \frac{9}{131072e}(1008 - 1008e - 20392e^2 - 39024e^3 - 22117e^4 + 31425e^5 + 28034e^6 + 31254e^7),$$

$$\beta_{21} = -\frac{9}{262144e}(144 - 144e - 25944e^2 - 31824e^3 - 61035e^4 - 3657e^5 + 124276e^6 + 54870e^7),$$

$$\beta_{22} = \frac{9}{131072e}(144 - 144e - 11288e^2 - 15120e^3 - 26067e^4 + 1527e^5 + 54198e^6 + 22794e^7),$$

$$\beta_{23} = -\frac{9}{262144e}(2160 - 2160e - 37416e^2 - 76464e^3 - 35333e^4 + 69561e^5 + 40188e^6 + 53226e^7),$$

$$\beta_{24} = -\frac{27}{1048576} e(-168 - 48e - 1880e^2 - 1848e^3 + 1387e^4 + 2436e^5),$$

$$\beta_{25} = \frac{27}{2097152} e(-1368 - 528e - 8360e^2 - 10728e^3 + 11797e^4 + 18876e^5),$$

$$\beta_{26} = -\frac{27}{1048576} e(-792 - 336e - 3304e^2 - 5256e^3 + 6941e^4 + 10716e^5),$$

$$\beta_{27} = \frac{27}{2097152} e(-552 - 240e - 2008e^2 - 3480e^3 + 4859e^4 + 7428e^5),$$