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Original Research Article

ANALYTICAL AND GRAPHYCAL SOLUTION OF ONE-DIMENSIONAL WAVE EQUATION USING MAPLE 18 SOFTWARE

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Abstract: This work provides the preliminary information for future research work on analytical and graphical solution of series ODEs and PDEs using maple18 software. The computations with the maple 18 software consist of solving procedure, animation, plotting of 2-D and 3-D graphs. This project work is considered as a blend of mathematics and computer science leading to powerful methods for solving complex problems on one-dimensional wave equation with d'Alembert formula. It is also considered for creating a mathematical model of wave problems and methods of solving the wave problems by implementing it on Maple worksheet which can be used in several undergraduate and graduate courses as well as solving new research problems.

Introduction: Many problems in various branches of mathematics, science and engineering often require cumbersome analytical computation of solving, plotting animating the graph of one dimensional wave equation. These equations are so difficult in many cases, even impossible to perform by hand and the need to solve these problems with fast and easy analytic computation, which has lead to the idea of using maple to extensively perform an analytical, graphical and animation of one-dimensional wave equations.

Historically, the problem of wave equations was studied by Jean le Round d' Alembert using a vibrating string such as that of a musical instrument. In 1746, d'Alembert discovered the one dimensional wave equation which is hyperbolic partial differential equation. It typically concerns a time variable **t** one or more partial variables $\mathbf{x_1, x_2, \ldots\ldots x_n}$ and a scalar function $\mathbf{U = U(x_1, x_2, \ldots. x_{nt})}$ wave defined as below.

$$diff(u(x,t), t\$2) = c^2 \cdot diff(u(x,t), x\$2);$$

$$\frac{\partial^2}{\partial t^2} u(x,t) = c^2 \left(\frac{\partial^2}{\partial x^2} u(x,t) \right)$$

Or in separation of variables form which is computed as

$$> U[tt] = C^2 U[xx];$$

$$U_{tt} = C^2 \, U_{xx}$$

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This was derived in class for small amplitude vibrations of a uniform string under a constant tension. The equation alone does not specify a solution but a unique solution is usually obtained by setting a problem with further conditions, such as initial conditions and another important class of problems called boundary conditions.

Since the wave equation is a linear second order PDE, given ant twice-differentiable functions of a single variable called f_1 and f_2

> $f[1](x+ct)$

$$f_1(x+ct)$$

And

> $f[2](x-c\cdot t);$

$$f_2(-c\,t+x)$$

In 18th century d'Alembert noted that the plus and minus signs in **x + ct** and **x - ct** indicate the direction of waves as **f_2(x - ct)** travels to the right then **f_1(x + ct)** travels to the left.

> $U[(f[1]f[2])]=0$

$$U_{f_1 f_2}=0$$

The equation is integrated to find the solution of wave equation by taking the form of a sum of a wave traveling to the right and one to the left. The **wave function** can be computed as below.

>

$U(x,t)=f[1](x+ct)+f[2](x-ct);$

$$U(x,t)=f_1(x+ct)+f_2(x-ct)$$

Or

> $U(x,t)=F(x+ct)+G(x-ct);$

$$U(x,t)=F(x+ct)+G(x-ct)$$

Where (**F** and **G**) or (**f_1** and **f_2**) are the arbitrary constants that can be determined from prescribed initial and boundary conditions.

The two initial conditions specified are

> $u(x,0)=f(x);$

$u(x,0)=f(x)$ is the initial displacement.

> $u(x,0)=g(x)$

$u(x,0)=g(x)$ is the initial velocity.

As c = speed

With which the d'Alembert formula was formed as

$$u:=(x,t)\to\frac{1}{2}f(x+c\,t)+f(x-c\,t)+\left(\frac{1}{2\,c}\right)\left(\int_{x-ct}^{x+ct}g(s)\,ds\right)$$

Which can be computed on Maple worksheet as below.

>

$u:=(x,t)\to\frac{1}{2}\cdot f(x+c\cdot t)+f(x-c\cdot t)+\frac{1}{2\cdot c}(int(g(s),s=(x-ct)..(x+ct)));$

$$u:=(x,t)\to\frac{1}{2}f(x+c\,t)+f(x-c\,t)+\left(\frac{1}{2\,c}\right)\left(\int_{x-ct}^{x+ct}g(s)\,ds\right)$$

Basic Features of Maple 18 Software

- It is fast in symbolic, numerical and interactive computation.
- It is accessible to large number of students and researchers.
- It is available for almost all operating systems.
- It is powerful programming language, intuitive syntax and easy to debug.
- It consist an extensive library of mathematical functions and specialized packages like **with(plots)**, **with(PDEtools)** e.t.c.

Methodology

Suppose the **ICs** are **u(x, 0) = 1/2 e(-x^2)** and **u_t (x,0) = e (-x^2)** for **c = 4** use d'Alembert computation to find the wave function **u (x , t).**

Solution

Start with the tools command **With(PDEtools)** on the worksheet and define the wave equation formula with any choice of abbreviation e.g **WaveEq or Wav.**

> $With(PDEtools):$

> $WaveEq:=diff(u(x,t),t,t)=c^2\cdot diff(u(x,t),x,x);$

$$WaveEq:=\frac{\partial^2}{\partial t^2}u(x,t)=c^2\left(\frac{\partial^2}{\partial x^2}u(x,t)\right)$$

The initial displacement and initial velocity are **u(x, 0) = 1/2 e(-x^2)** and **u_t (x,0) = e (-x^2)** with the speed **c= 4.**

Compute the command as **pdsolve** which means to find the partial differential equation with d'Alembert formula.

> $pdsolve(\{WaveEq, u(x, 0) = f(x), D[2](u)(x, 0) = g(x)\});$

$$u(x, t) = \frac{1}{2}\,\frac{1}{c}\left(f(-c\,t+x)\,c + f(c\,t+x)\,c - \left(\int_0^{-c\,t+x} g(x1)\,\mathrm{d}x1\right) + \int_0^{c\,t+x} g(x1)\,\mathrm{d}x1\right)$$

Use dA (d'Alembert) to combine into single term

> $dA := IntegrationTools{:-}Combine(\%);$

$$dA := u(x, t) = \frac{1}{2}\,\frac{1}{c}\left(-\left(\int_{c\,t+x}^{-c\,t+x} g(x1)\,\mathrm{d}x1\right) + f(-c\,t+x)\,c + f(c\,t+x)\,c\right)$$

Use **eval** to evaluate the expression in dA

> $eval\left(dA, \left\{c = 4, f = \left(x \rightarrow \frac{1}{2}\cdot \exp(-x^2)\right), g = (x \rightarrow \exp(-x^2))\right\}\right)$

$$u(x, t) = -\frac{1}{8}\int_{4\,t+x}^{-4\,t+x} \mathrm{e}^{-x1^2}\,\mathrm{d}x1 + \frac{1}{4}\,\mathrm{e}^{-(-4\,t+x)^2} + \frac{1}{4}\,\mathrm{e}^{-(4\,t+x)^2}$$

Use the command value(%) to evaluate the inert function of **dA** by defining it as **dA1** or with any other abbreviation.

> $dA1 := value(\%);$

$$dA1 := u(x, t) = \frac{1}{16}\sqrt{\pi}\,\mathrm{erf}(4\,t+x) + \frac{1}{16}\sqrt{\pi}\,\mathrm{erf}(4\,t-x) + \frac{1}{4}\,\mathrm{e}^{-(-4\,t+x)^2} + \frac{1}{4}\,\mathrm{e}^{-(4\,t+x)^2}$$

$> f := x \rightarrow \frac{1}{2} \cdot \exp(-x^2);$

$$f := x \rightarrow \frac{1}{2}\, e^{-x^2}$$

$plot(f(x), x = -20..20);$

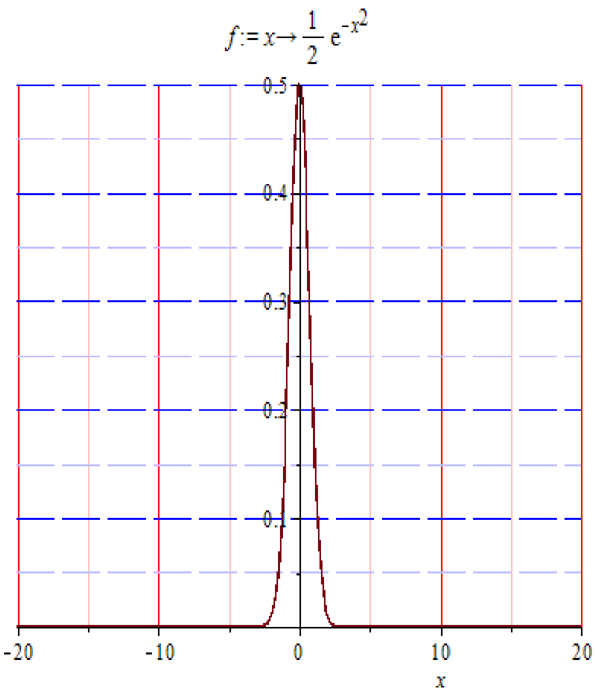

fig 1.

Method of Plotting Graph f(x)

When $f := x \rightarrow \frac{1}{2} \cdot \exp(-x^2);$

(a) Then enter $plot(f(x), x = -20..20);$ on the work sheet.

(b) Immediately the graph appears, move the cursor and click on the graph.

(c) Move the mouse to **change gridlines properties** on the standard toolbar and left click.

(d) The **Axis Gridlines Properties** displays with the **Vertical** & **Horizontal** Option .select **Show Gridlines,** your choice of **color** and click **apply** in both options.

(e) Finally click the **OK.**

$> g := x \rightarrow \exp(-x^2);$

$$g := x \rightarrow e^{-x^2}$$

$plot(g(x), x = -20..20);$

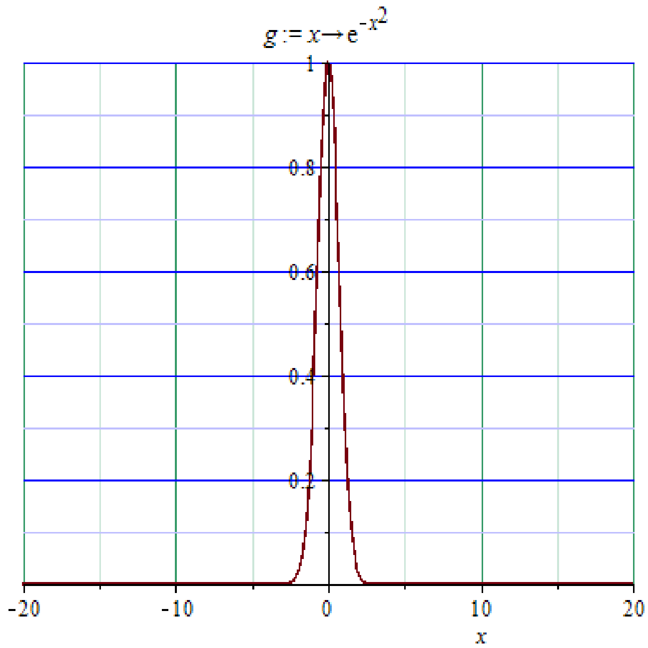

Fig 2.

METHOD OF PLOTTING GRAPH g(x)

When $g := x \rightarrow \exp(-x^2);$

(a) Then enter $plot(g(x), x = -20..20);$ on the work sheet.

(b) Immediately the graph appears, click on the graph.

(c) Move the mouse to **change gridlines properties** on the standard toolbar and left click.

(d) On **Axis Gridlines Properties** with the **Vertical** Option select **Show Gridlines,** your choice of **color** and click **apply**.

(e) Select the **Horizontal** option and follow the same procedure

(f) Finally click the **OK.**

$$plot3d\left(\cos(u(x, t)), x = -40 \cdot \mathrm{Pi}..150, t = -\frac{76}{\mathrm{Pi}}..\frac{20}{\mathrm{Pi}}, axes = boxed\right)$$

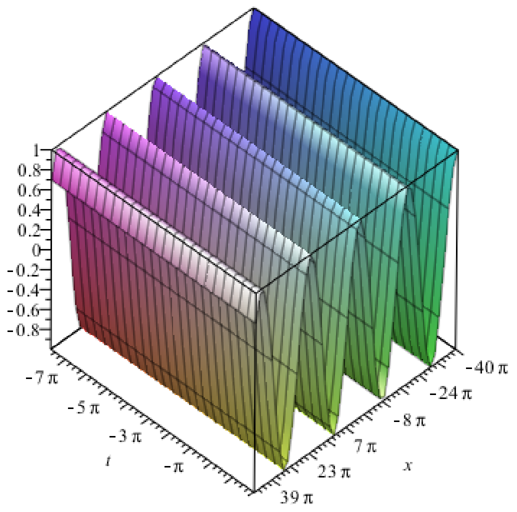

Fig 3

Method of Plotting 3-D Graph Of Solution Cos u (x , t) .

Since

$$dA1 := u(x,t) = \frac{1}{16}\sqrt{\pi}\,\mathrm{erf}(4t+x) + \frac{1}{16}\sqrt{\pi}\,\mathrm{erf}(4t-x) + \frac{1}{4} + \frac{1}{4}\,\mathrm{e}^{-(4t+x)^2};$$

(a) Then enter

$$plot3d\left(\cos(u(x,t)),\, x=-40\cdot\mathrm{Pi}..150,\, t=-\frac{76}{\mathrm{Pi}}..\frac{20}{\mathrm{Pi}},\, axes=boxed\right);$$

on the work sheet.

(b) Immediately the graph appears, move the cursor and click on the graph.

(c) Move the mouse to **change axis properties** on the standard toolbar and left click.

(d) On **Axis Properties** with the **X-axis** Option select **Log mode,** your choice of **color** and click **apply**.

(e) Select the **Y** and **Z-axis** option and follow the same procedure

(f) Finally click the **OK** as the graph changes to the object below

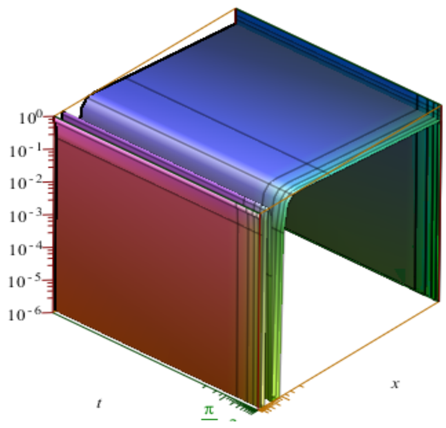

Fig 4

Animation of 3-D Graph

Apply d'Alembert formula to form the animation and 3-D graph of a wave function given by the initial conditions **u (x, 0)=1/2e(-x^2),ut(x,0)=0** with **c = 4**.

> $with(PDEtools)$:

> $WaveEq := diff(u(x,t), t, t) = c^2\cdot diff(u(x,t), x, x);$

$$WaveEq := \frac{\partial^2}{\partial t^2}u(x,t) = c^2\left(\frac{\partial^2}{\partial x^2}u(x,t)\right)$$

> $pdsolve(\{WaveEq, u(x,0)=f(x), \mathrm{D}[2](u)(x,0)=g(x)\});$

$$u(x,t) = \frac{1}{2}\,\frac{1}{c}\left(f(-c\,t+x)\,c + f(c\,t+x)\,c - \left(\int_0^{-c\,t+x} g(x1)\,\mathrm{d}x1\right) + \int_0^{c\,t+x} g(x1)\,\mathrm{d}x1\right)$$

> $dA := IntegrationTools\text{:-}Combine(\%);$

$$dA := u(x, t) = \frac{1}{2} \frac{-\left(\int_{c\,t+x}^{-c\,t+x} g(x1)\,\mathrm{d}x1\right) + f(-c\,t+x)\,c + f(c\,t+x)\,c}{c}$$

$$> eval\left(dA, \left\{c = 4, f = \left(x \to \frac{1}{2} \cdot \exp(-x^2)\right), g = 0\right\}\right);$$

$$u(x, t) = -\frac{1}{8} \int_{4\,t+x}^{-4\,t+x} 0\,\mathrm{d}x1 + \frac{1}{4}\,\mathrm{e}^{-(-4\,t+x)^2} + \frac{1}{4}\,\mathrm{e}^{-(4\,t+x)^2}$$

$$> dA1 := value(\%);$$

$$dA1 := u(x, t) = \frac{1}{4}\,\mathrm{e}^{-(-4\,t+x)^2} + \frac{1}{4}\,\mathrm{e}^{-(4\,t+x)^2}$$

$with(plots):$

$$plot3d\left(\frac{1}{4}\,\mathrm{e}^{-(-4\,t+x)^2} + \frac{1}{4}\,\mathrm{e}^{-(4\,t+x)^2}, x = -5..5, t = -5..5\right);$$

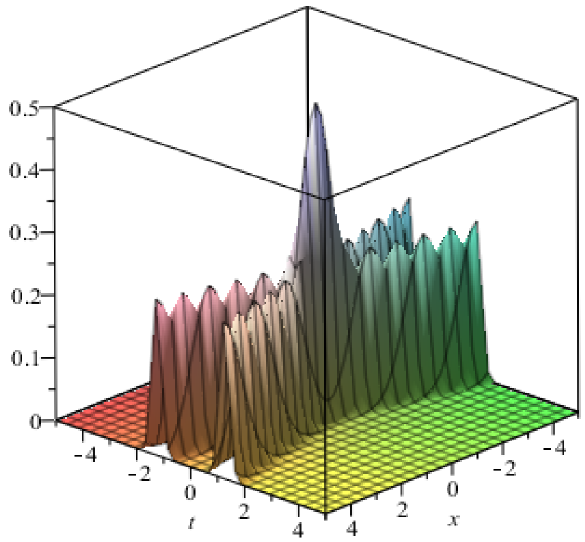

Fig 5

METHOD OF PLOTTING 3-D GRAPH OF SOLUTION u (x , t) .

When

$$dA1 := u(x, t) = \frac{1}{4}\,\mathrm{e}^{-(-4\,t+x)^2} + \frac{1}{4}\,\mathrm{e}^{-(4\,t+x)^2}$$

then

(a) Enter $plot3d\left(\frac{1}{4}\,\mathrm{e}^{-(-4\,t+x)^2} + \frac{1}{4}\,\mathrm{e}^{-(4\,t+x)^2}, x = -5..5, t = -5..5\right);$ on the work sheet.

(b) Immediately the graph appears, click on the graph.

(c) Move the mouse to **change axis properties** on the standard toolbar and left click.

(d) On **Axis Properties** with the **X-axis** Option select **Log mode,** your choice of **color** and click **apply**.

(e) Select the **Y** and **Z-axis** option and follow the same procedure

(f) Finally click the **OK** as the graph changes to the object below.

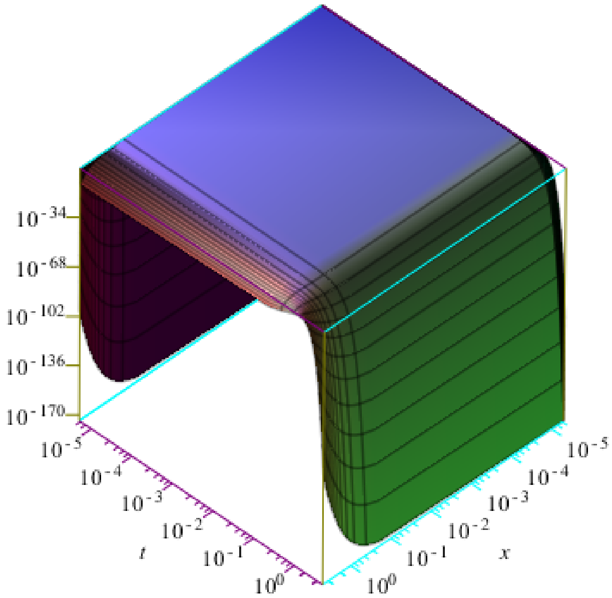

Fig 6

$$plots_{animate}\left('plots_{complexplot3d}', \left[\frac{1}{4}\,e^{-(-4t+x)^2} + \frac{1}{4}\,e^{-(4t+x)^2}, x = -15 - I..15 + I, labels = [x, "", ""]\right], t = 0..10\right);$$

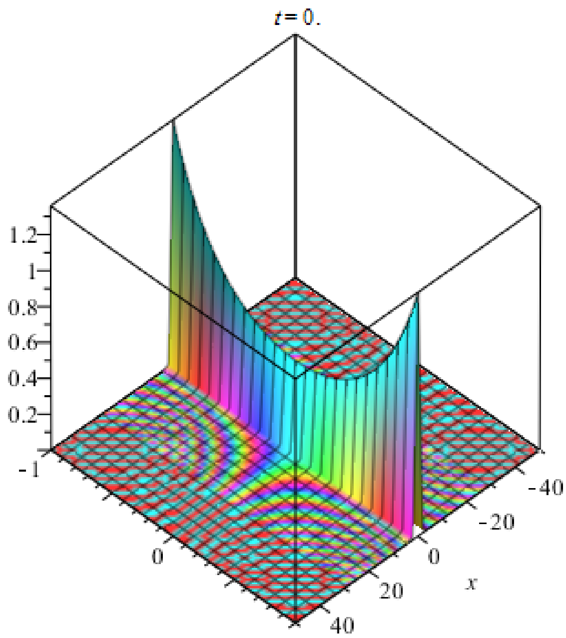

Fig 7

Move the cursor to the graph, right click the mouse and select **animation** and **play** on the menu drop down list.
Plot the 3-D graph of the pulse begin at **t = 0..10** when **u = (x, 0) = e-(x-3)^2** and $\mathbf{u_t}$ **= (x, 0) = 0** at **c=4**.

> *restart*;
> *with*(*PDEtools*) :
> *WaveEq* := *diff*(*u*(*x*, *t*), *t*, *t*) = c^2·*diff*(*u*(*x*, *t*), *x*, *x*);

$$WaveEq := \frac{\partial^2}{\partial t^2} u(x,t) = c^2 \left(\frac{\partial^2}{\partial x^2} u(x,t) \right)$$

> *pdsolve*({*WaveEq*, *u*(*x*, 0) = *f*(*x*), D[2](*u*)(*x*, 0) = *g*(*x*)});

$$u(x,t) = \frac{1}{2}\,\frac{1}{c}\left(f(-c\,t+x)\,c + f(c\,t+x)\,c - \left(\int_0^{-c\,t+x} g(x1)\,dx1 \right) + \int_0^{c\,t+x} g(x1)\,dx1 \right)$$

> *dA* := *IntegrationTools*:-*Combine*(%);

$$dA := u(x,t) = \frac{1}{2}\,\frac{-\left(\int_{c\,t+x}^{-c\,t+x} g(x1)\,dx1 \right) + f(-c\,t+x)\,c + f(c\,t+x)\,c}{c}$$

> $eval\left(dA, \left\{c = 4, f = \left(x \rightarrow \frac{1}{2}\cdot \exp-(x-3)^2\right), g = 0\right\}\right);$

$$u(x,t) = -\frac{1}{8}\int_{4\,t+x}^{-4\,t+x} 0\,dx1 + \frac{1}{2}\exp - \frac{1}{2}(-4\,t+x-3)^2 - \frac{1}{2}(4\,t+x-3)^2$$

> *dA1* := *value*(%);

$$dA1 := u(x,t) = \frac{1}{2}\exp - \frac{1}{2}(-4\,t+x-3)^2 - \frac{1}{2}(4\,t+x-3)^2$$

with(*plots*) :

$$plots_{animate}\left('plot3d', \left[\frac{\sin\left(\frac{1}{2}\exp - \frac{1}{2}(-4t+x-3)^2 - \frac{1}{2}(4t+x-3)^2\right)}{\pi}, \exp = 0..25, x = 0..3, labels = [\exp, x, ""]\right], t = 0..10\right);$$

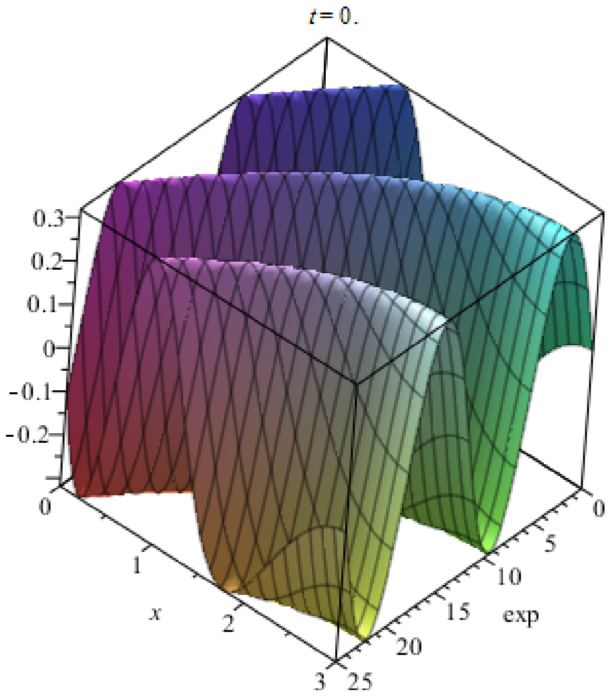

Fig 8

Move the cursor to the graph, right click the mouse and select **animation** and **play** on the menu drop down list.
formula method, it is a solving procedure and animation of 3-D graph of wave function **u(x,t).**

CONCLUTION

Using Maple 18 software in solving one-dimensional wave equation is an exposure and development to the computation of d'Alembert

SUMMARY

This project work shows the procedure in solving one-dimensional wave equation with d'Alembert formula using **Maple** 18 software

(1) The word **Restart** is used to clear the internal memory of previous solving that may affect the new solving.
(2) Wave formula is defined on the worksheet as

$$> WaveEq := diff(u(x,t), t, t) = c^2 \cdot diff(u(x,t), x, x);$$

$$WaveEq := \frac{\partial^2}{\partial t^2} u(x,t) = c^2 \left(\frac{\partial^2}{\partial x^2} u(x,t)\right)$$

(3) The calling sequence for computing d'Alembert formula is

$$> pdsolve(\{WaveEq, u(x, 0) = f(x), D[2](u)(x, 0) = g(x)\});$$

$$u(x, t) = \frac{1}{2}\frac{1}{c}\left(f(-c\,t + x)\,c + f(c\,t + x)\,c - \left(\int_0^{-c\,t + x} g(x1)\,dx1\right) + \int_0^{c\,t + x} g(x1)\,dx1\right)$$

(4) The calling sequence for solving with d'Alembert formula is

$$eval\left(res, \left\{c = 4, f = \left(x \rightarrow \frac{1}{2}\cdot \exp(-x^2)\right), g = \left(x \rightarrow \exp(-x^2)\right)\right\}\right);$$

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